

Research on GPS Derived Orthometric Height: A Case Study

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1. INTRODUCTION

This paper discusses three different methods in deriving orthometric height from GPS ellipsoidal height, that is, the conicoid fitting method (CFM) frequently used in China, the Geoid Modeling GEOID99 developed by National Geodetic Survey (NGS) in USA, and the recently developed new method – the “Hu-method”, based on neural network concept.

A brief summary of the theoretical aspects of the different methods is given first. A practical engineering project is then used to analyze the efficacy of these methods. It is shown on this project that the NGS Geoid Model GEOID99 gives an accuracy of about 26 mm for GPS-derived orthometric height. The conicoid fitting method (CFM) gives an accuracy of about 85 mm. And finally, the newly developed “Hu-method” based on neural network concept produces an accuracy of about 18 mm. Therefore, the new method produced better results than either the CFM or the GEOID99 in deriving orthometric height from GPS height. Some suggestions and recommendations are drawn.

2. THE CONICOID FITTING METHOD (CFM)

To derive orthometric height (H_{Nor}) from GPS ellipsoidal height (H_{GPS}), the conicoid fitting method (CFM) is frequently used in China. The main idea of CFM is to design a set of control points whose H_{Nor} and H_{GPS} are known, and then the elevation anomaly ξ (the difference between H_{GPS} and H_{Nor}) is modeled by a polynomial of second degree, as follows (Liu et al. 1996):

$$\xi(x, y) = a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 \quad (1)$$

Where x, y are the horizontal coordinates of a control point, and a_0, a_1, \dots, a_5 the unknown coefficients.

A minimum of six control points with known H_{GPS} and H_{Nor} are needed.

3. THE HU-METHOD BASED ON NEURAL NETWORK CONCEPT

3.1 The structure of the BP neural networks

The structure of BP neural networks is shown in Figure 1 below. It can be divided into five layers. For an ordinary engineering application, the input and output transformation layers are needed because the Sigmoid standard active function $f(x)$ ranges from 0 to 1 (cf. Hu, 2001b).

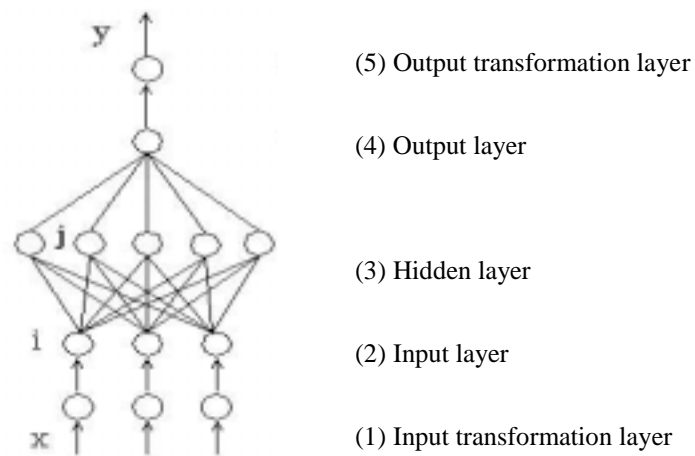


Figure 1: A single hidden layer model of BP networks

For detailed computation formula of the BP neural networks, the readers are referred to Yuan (1999) and Hu (2001b).

3.2 The concept of the Hu-Method

The procedure of the Hu-method is as follows (Hu et al. 2003):

- 1) Assume there are n points, of which values of H_{GPS} and H_{Nor} of n_1 points are known, and n_2 ($n_2=n-n_1$) point values of H_{Nor} need to be calculated. The requirement of this method is that n_1 must be greater than 8.
- 2) Based on the n_1 points, elevation anomaly (ξ) can be calculated for all points by CFM.
- 3) Calculate the error of elevation anomaly of CFM for all n_1 points as follows

$$\Delta\xi = \xi_0 - \xi \quad (2)$$

Where $\xi_0 = H_{GPS} - H_{Nor}$, representing the known value of elevation anomaly, and the ξ is the elevation anomaly by CFM.

- 4) Use the information of the above n_1 points ($x_i, y_i, \xi_i; \Delta\xi_i; i = 1, 2, \dots, n_1$) as a set of samples for

study. The BP network is trained by these samples. (Notice: x_i, y_i, ξ_i are the three units in the input layer; $\Delta \xi_i$ is the one unit in the output layer.)

- 5) The error of elevation anomaly ($\Delta \xi$) can be calculated by the trained BP network for all n_2 points.

The orthometric height can be calculated by

$$H_{Nor} = H_{GPS} - \xi_0 = H_{GPS} - (\xi + \Delta \xi) \quad (3)$$

Where ξ is calculated by the CFM and $\Delta \xi$ by the Neural Network Method (NNM)

4. A CASE STUDY

In order to study the efficiency of the different methods in deriving orthometric height from GPS ellipsoidal height, a practical GPS survey project is used. The project is located in central Illinois, U.S.A. There are a total of 215 GPS network stations on this project covering about 28000 km². The GPS survey was conducted by ASC American Surveying Consultants, P.C. with three Trimble dual frequency geodetic receivers with L1/L2 geodetic antennas (one Trimble 4800 and two Trimble 4700 GPS receivers), following the NGS Order C 1 accuracy standard (cf. Kuang et al., 2002).

On this project, there are a total of 63 NGS benchmarks tied into the network. Thus, one needs to derive orthometric heights for the remaining 152 network points by combining GPS heights and the known benchmarks. Fig. 2 below shows the distribution of the 215 network stations where, in order to distinguish between the benchmarks and non-benchmarks, the point numbers of the 63 benchmarks are shown, and they are denoted by a circle. The remaining 152 stations (non-benchmarks) are denoted by a cross on the map.

Table 1: Comparison between Benchmark Elevations and GPS derived Elevations from NGS Geoid99 Model

Unites = Meters; H1 = The known Benchmark elevation; H2 = The elevation by GEOID99 Model

Point#	GPS Height	H1	H2	Delta (H1-H2)	Point#	GPS Height	H1	H2	Delta (H1-H2)
19	185.445	217.412	217.429	-0.017	704	205.795	237.765	237.794	-0.029
30	169.608	201.869	201.833	0.036	719	160.968	193.860	193.848	0.012
33	171.502	203.643	203.646	-0.003	721	193.668	225.685	225.716	-0.031
44	204.763	236.976	237.036	-0.060	723	164.227	197.076	197.061	0.015
57	191.983	224.253	224.277	-0.024	728	171.049	203.836	203.827	0.009
80	173.757	206.049	206.072	-0.023	734	185.016	217.749	217.739	0.010
90	170.150	202.480	202.490	-0.010	738	176.908	209.616	209.605	0.011
96	173.223	205.583	205.573	0.010	752	171.995	204.073	204.082	-0.009
104	166.145	198.539	198.531	0.008	774	168.198	200.328	200.328	0.000
225	153.791	186.301	186.308	-0.007	780	123.927	156.988	156.978	0.010
243	165.142	197.740	197.720	0.020	815	218.018	250.390	250.413	-0.023
249	154.638	187.178	187.175	0.003	855	181.232	213.550	213.554	-0.004
254	154.827	187.246	187.291	-0.045	874	187.944	220.516	220.540	-0.024
284	153.171	185.702	185.692	0.010	974	211.710	243.829	243.835	-0.006
289	149.412	181.976	181.972	0.004	1010	227.662	259.737	259.760	-0.023
294	146.302	178.928	178.913	0.015	1036	182.435	214.770	214.774	-0.004
304	145.924	178.542	178.596	-0.054	1051	189.246	221.632	221.661	-0.029
342	174.975	207.332	207.350	-0.018	1294	194.186	226.420	226.442	-0.022
390	154.952	187.813	187.810	0.003	1498	159.303	192.557	192.512	0.045
419	122.368	155.079	155.083	-0.004	1513	158.079	191.224	191.175	0.049
432	185.792	218.002	217.978	0.024	1537	157.441	190.031	190.046	-0.015
444	212.404	244.481	244.483	-0.002	1548	177.673	210.644	210.600	0.044
470	195.766	227.845	227.895	-0.050	1553	174.385	207.364	207.352	0.012
507	182.618	215.750	215.803	-0.053	1675	221.009	253.041	253.056	-0.015
536	210.534	242.466	242.506	-0.040	1683	208.526	240.405	240.459	-0.054
606	234.031	267.029	267.024	0.005	1687	209.756	241.695	241.741	-0.046
637	188.242	221.264	221.252	0.012	1987	171.521	203.974	203.971	0.003
663	196.126	229.129	229.079	0.050	1999	168.769	201.257	201.275	-0.018
668	192.437	224.561	224.559	0.002	7012	187.529	220.028	219.998	0.030
675	178.488	211.453	211.438	0.015	7013	178.225	211.213	211.247	-0.034
683	169.038	202.002	201.994	0.008	9450	176.031	208.654	208.649	0.005
691	164.102	197.043	197.051	-0.008	Mean Square Error =				0.026

4.2 The CFM Method

With the CFM method, 38 of the 63 NGS benchmarks are taken as the study group, and the remaining 25 points as the test group. They are denoted by squares and circles, respectively, in Fig. 3 below. Note that, for the purpose of clarity, the non-benchmarks are omitted in Fig. 3.



Fig. 3: The Study and Test Benchmark Points for CFM

The unknown coefficients a_0, a_1, \dots, a_5 in equation (1) can be obtained using the study group by the CFM, and then the elevation anomaly (ξ) of all the 215 points can be computed by equation (1). The results of CFM are summarized in Table 2 below.

Table 2: The Results of CFM

Study group	Results	$n_1=38$	$m_1= \pm 85.7\text{mm}$
	Note	m_1 =The mean square error of study group	
Test group	Results	$n_2=25$	$m_2= \pm 85.1\text{mm}$
	Note	m_2 =The mean square error of test group	
All the 63 points	Results	$n=63$	$m= \pm 85.3\text{mm}$
	Note	m =The mean square error of all the 63 points	
The unknown coefficients		$a_0= 43.193026, a_1= -0.64228055, a_2= -0.14915228,$ $a_3= 0.01229524, a_4= -1.0370501\text{E-}03, a_5=2.5932532\text{E-}03.$	

4.3 The Hu-Method

Following the procedures of the Hu-method discussed above, the results are listed in Table 3 below.

Table 3: The Results of the Hu-Method (Units = Meters)

Point#	Benchmark Elevation (H1)	Elevation by Hu-method (H3)	Delta (H1-H3)	Point#	Benchmark Elevation (H1)	Elevation by Hu-method (H3)	Delta (H1-H3)
19	217.412	217.412	0.000	719	193.860	193.889	-0.029
30	201.869	201.886	-0.017	721	225.685	225.701	-0.016
33	203.643	203.638	0.005	723	197.076	197.092	-0.016
44	236.976	236.971	0.005	728	203.836	203.836	0.000
57	224.253	224.275	-0.022	734	217.749	217.761	-0.012
80	206.049	206.054	-0.005	738	209.616	209.638	-0.022
90	202.480	202.485	-0.005	752	204.073	204.068	0.005
96	205.583	205.597	-0.014	774	200.328	200.338	-0.010
104	198.539	198.544	-0.005	780	156.988	156.987	0.001
225	186.301	186.306	-0.005	815	250.390	250.385	0.005
243	197.740	197.746	-0.006	855	213.550	213.547	0.003
249	187.178	187.172	0.006	874	220.516	220.520	-0.004
254	187.246	187.188	0.058	974	243.829	243.832	-0.003
284	185.702	185.721	-0.019	1010	259.737	259.742	-0.005
289	181.976	181.997	-0.021	1036	214.770	214.791	-0.021
294	178.928	178.973	-0.045	1051	221.632	221.636	-0.004
304	178.542	178.539	0.003	1294	226.420	226.425	-0.005
342	207.332	207.326	0.006	1498	192.557	192.562	-0.005
390	187.813	187.808	0.005	1513	191.224	191.262	-0.038
419	155.079	155.085	-0.006	1537	190.031	189.975	0.056
432	218.002	218.007	-0.005	1548	210.644	210.678	-0.034
444	244.481	244.512	-0.031	1553	207.364	207.358	0.006
470	227.845	227.842	0.003	1675	253.041	253.028	0.013
507	215.750	215.755	-0.005	1683	240.405	240.400	0.005
536	242.466	242.470	-0.004	1687	241.695	241.709	-0.014
606	267.029	267.034	-0.005	1987	203.974	203.974	0.000
637	221.264	221.280	-0.016	1999	201.257	201.259	-0.002
663	229.129	229.174	-0.045	7012	220.028	220.033	-0.005
668	224.561	224.564	-0.003	7013	211.213	211.208	0.005
675	211.453	211.456	-0.003	9450	208.654	208.648	0.006
683	202.002	201.994	0.008				
691	197.043	197.039	0.004				
704	237.765	237.763	0.002				
Mean Square Error =							<u>0.018</u>

Analysis of the above results demonstrates that the Hu-method produced better results than either the GEOID99 or the CFM method in deriving orthometric height from GPS ellipsoidal height on this project.

5. CONCLUDING REMARKS

The Hu-method discussed in this paper can be used to detect the model error of CFM with the help of the neural network concept (cf. Hu et al. 2001a). This can be explained by the BP network structure of Hu-method: one parameter ξ calculated by CFM is in the input layer, and the parameter of the output layer is the difference of elevation anomaly ($\Delta\xi = \xi_0 - \xi$). Compared with CFM and the GEOID99 Model, the Hu-method may produce more accurate results in deriving orthometric height from GPS ellipsoidal height.

In general, the disadvantage of the method based on neural networks is that the results are unstable and the final results are largely influenced by the initial weights W_{ji}^0 (Yuan, 1999). However, the BP structure of the Hu-method discussed in this paper is not affected the initial weights W_{ji}^0 (cf. Hu, 2001b; Hu et al. 2003). Thus, it is stable in calculating $\Delta\xi$ (the error of the elevation anomaly), and it is therefore recommended for use in practice.

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